Mathematical modeling of seismic isolators

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ABSTRACT

Base isolation is a relatively new design strategy used to protect constructed facilities from seismic shaking. It has yet to be validated by satisfactory response being achieved in strong earthquakes. Also adequate methods of predicting, at the design stage, the degrading properties of isolators particularly under biaxial horizontal motion are still under development. This paper summarizes current design practices in which an isolator is treated as an equivalent elastic column at the base of a building and, for the analysis of concurrent horizontal orthogonal motions as two columns sharing the same space concentrically, one bending in one horizontal direction and the other in the perpendicular horizontal direction. Various sophisticated analysis techniques including finite element representations of individual isolators are reviewed and the need for an accurate model incorporating damping and stiffness degradation is emphasized. Progress in the development of such a model is reported.

INTRODUCTION

The principle of reducing the dynamic effects of seismic ground motions by providing a building with a flexible lower storey was popular at one stage of the evolution of earthquake resistant design. However the poor behavior of many flexible first-storey buildings emphasized the difficulties in avoiding catastrophic failure at zones of sudden change of stiffners and strength in the load path. More recently the concept of genuine seismic isolation using a damped flexible mechanism between the ground and the bottom of a structure has received considerable attention (Kelly 1986). The main concepts of base isolation are to lower the fundamental frequency of the structure below the predominant frequencies of the seismic excitation and to provide a mechanism for energy dissipation within the isolator.

In simplistic terms seismic isolation can be thought to be a trade-off of acceleration for displacement. The inertia loads on an isolated structure will be lowered, as also will be the interstorey drifts, at the expense of increased movement at the interface where the isolators are installed. Clearly seismic isolation is not applicable to all situations. Whereas it may very well

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be advantageous in the case of a constructed facility on a firm site, where the excitation from earthquakes will be of relatively high frequency, it is likely to be unsuited for application to soft sites with characteristic long period seismic responses.

Significant development of seismic isolation has occurred in the last decade as the result of advances in several areas. An improved understanding of the effects of site foundation conditions on incoming seismic waves, together with a much enlarged data base of earthquake records, has prompted more reliable predictions of site specific seismic movements. Advances in computer based mathematical modelling have permitted more comprehensive analyses to be undertaken at the design stage and, probably of greatest importance, the development of a variety of isolator units including laminated bearings using high-damping elastomer layered between metal plates has resulted in components of practical applicability being available.

New buildings incorporating base isolation have been completed in Japan, New Zealand and the U. S. A.(Anderson 1990). Bridges have been isolated, notably in New Zealand, as have nuclear power stations in France and South Africa. Some existing bridges and buildings have been retrofitted with base isolators as a means of reducing the earthquake forces applied to understrength structures. Clearly an increasing awareness of the potential benefits of seismic isolation is reflected in this activity. Future applications will be determined largely by the degree of field success enjoyed by existing installations in mitigating the effects of strong motion earthquakes and by the availability of analysis and design procedures which will ensure economical and reliable configurations.

CURRENT DESIGN CONSIDERATIONS

Elementary design considerations of base isolators can commence with an awareness that elastomeric bridge bearings may be sized (AASHTO 1983) using an allowable compressive stress of 1,000 to 2,000 p.s.i. whereas failure is expected at an order of magnitude greater pressure. Once an approximate plan area, A, is selected the unidirectional horizontal stiffness, K, of a simple bearing can be predicted, when G is the shear modulus and H is the height of the rubber in the bearing as:

$$K = \frac{GA}{H} \tag{1}$$

Practical considerations of overall stability and the need to provide relatively high vertical stiffness require that most base isolators comprise alternate layers of steel shims and elastomeric material. When the elastomer layers are thin relative to the bearing diameter, the bearings are referred to as having high shape factor (HSF). The shape factor S is defined as the compressive area divided by the circumferential area free to bulge in a single layer. For circular bearings, when D is the diameter of the steel shim and t is the thickness of one layer of elastomer,

$$S = \frac{D}{4t} \tag{2}$$

Since the stiffness of the metal shims is so much greater than that of the layer of elastomers, the horizontal stiffness equation (1) may be modified for a bearing having n layers of elastomer each of thickness t to be

$$K = \frac{GA}{nt} \tag{3}$$

Derham (1982) has proposed that the vertical stiffners K_v of bearings having shape factors of about 10 (i.e. medium shape factors) can be determined when E_c, the effective compression modulus equals $5.6GS^2$, to be

$$K_v = \frac{E_c A}{nt} \tag{4}$$

Kelly et al. (1990) have suggested modifications of this expression for high shape factor bearings.

Several variations on standard cylindrical or square laminated isolators have been developed including those with lead plugs through the center, to increase energy dissipation, and initially slack vertical chains connecting top and bottom plates, to limit the sideways distortion. This last development appears to have been prompted, in some degree at least, by awareness of the potential for roll-out and instability of bearings under the combined action of significant vertical loads and very large (greater than 100%) horizontal shear strains. Such high strains will result in a reduction of the effective area resisting vertical loads and contribute to the measured non-linear behavior at high strains of tested bearings. Additional contributions to observed non-linear characteristics are provided by the inelastic properties of the elastomer and, in the case of the isolators with lead plug inserts, by the yielding of this component.

Initial designs of seismic isolation systems may be undertaken on the basis of the assumptions of extended elastic behavior summarized above. Refinements may involve ETABS (Wilson 1975) type analyses and even non-linear investigations using coding such as ANSR (Oughourlian 1982). However current limitations on the available data base of non-linear material characteristics, including the properties of the yield surfaces of a three dimension model of an isolator, seriously restrict viable inelastic analyses. Consequently current design techniques are conservative to ensure an adequate factor of safety against collapse or instability of elastomeric bearings. This situation is unlikely to change until results are available of definitive experimental and theoretical studies of the mechanics of deformation of such components.

Several researchers have attempted to extend the simplistic models of base isolators to more valid representations. One such effort (Yasaka 1989) depicts the behavior of an isolator subject to biaxial shearing forces with either a Kinematic Hardening model or a Multiple Shear Spring model. The first of these is limited to bi-linear hysteretic characteristics whereas the actual properties of an isolator are more complicated. This restriction prompted the development of the second model in which several inelastic spring elements are equally spaced radially. The restoring force in each spring element q may be expressed

$$q = q(E, \dot{E}) \tag{5}$$

Where E and E of an element are respectively the deflection and rate of deflection.

The deflection E_i of an element i is given by

$$E_i = u_x \cos\theta ; u_y \sin\theta_i \tag{6}$$

where u_x and u_y are the deflection in each of the x and y directions and θ_i is the direction angle of the i th element. Also

$$\theta_i = (\frac{\pi}{N})_i \tag{7}$$

where N is the number of spring elements. The greater is N, the more accurate is the model.

The total restoring forces F_x on F_y along the x and y axes are also functions of the deflections and deflection rates viz:

$$F_x = \sum_{i=1}^{N} q(E_i, \dot{E}_i) \cos\theta_i \tag{8}$$

and

$$F_y = \sum_{i=1}^{N} q(E_i, \dot{E}_i) \sin\theta_i \tag{9}$$

Substitution of equation (6) into equations (8) and (9) allows expression of the restoring forces in terms of the orthogonal deflections u_x and u_y i.e.

$$F_x = \sum_{i=1}^{N} q(u_x \cos\theta_i, \dot{u} \cos\theta_i) \cos\theta_i \tag{10}$$

The function of q which satisfies equations (5) and (10) can be found if the uniaxial restoring force F_x is known. The Kinematic hardening and Multiple Sheer spring models have been compared using for different types of loading namely uniaxial, circular, oval and eight shaped. Good correlation was obtained providing N was larger than six.

Despite the fact that modelling is complicated by inherent material and geometric properties at the large deformations experienced, coupled with the difficulty of evaluating the constitutive property of a material such as rubber which varies from sample to sample (Mark 1982), attempts have been made to undertake finite element analyses. The composite layered system can be treated as a series of discrete units, with each element containing only one type of material, or composite with the system represented as an equivalent homogeneous orthotropic continuum. The discrete analysis necessitates detailed geometry but allows direct calculation of the local stresses and strains at the layer interfaces and the edges of the bearings. The composite approach is computationally more economic at the expense of providing less detailed results.

Seki (1987) has reported analyses undertaken at the Technical Research Laboratory of the Bridgestone Corporation. Strip biaxial testing provided details of the strain energy density function used to set up a two dimensional finite element mesh. It is claimed that the analyses agree well with experimental results.

In recognition that the constitutive laws for natural and synthetic rubber are not established reliably, earlier work (Lim 1987) at the University of California, Davis, modelled elastomer as nonlinear elastic material which is adequate except in cases where loads are cyclic and damping is important. More recent research (Herrmann 1988) has resulted in the development of a composite three dimensional finite element procedure based on an equivalent homogeneous continuum. Both geometric and material non-linearities are allowed for and a unit cell concept is utilized. Verification has been undertaken by comparing results of the finite element predictions with those of discrete analyses of simple configurations and of experimental measurements of actual bearing behavior. However Herrmann (1988) stresses that residual uncertainty exists regarding the adequacy of the non-linear elastic characterization used for the elastomer. He also emphasizes that, in the case of cyclic loading of bearings, analyses need to incorporate considerations of material inelasticity.

Several finite element methods for modelling layered elastomeric isolator bearings have been developed at the Argonne National Laboratory (Kulak 1989). The analysis of low to medium shape factor bearings is tackled by a discrete approach in which there is only one material per element and each steel and elastomer layer is represented by several elements through their thickness. It is envisaged that future development will involve a composite element such as Herrmann's and will be better suited to the analysis of high shape factor components. Kulak and Wang (1989) note the present limitation of only a bi-linear elasto-plastic constitutive equation being available in the non-linear spring element for simulating the horizontal response of the isolator and indicate that efforts are being made to develop a constitutive relation appropriate to the response of a laminated elastomer bearing.

OBJECTIVES OF PRESENT WORK

In order that the state-of-understanding of the mechanics of the deformation of elastomeric bearings at high shear strain can be advanced several important aspects need to be addressed.

Improved understanding is necessary of the distribution of stress and strain under both tension and compression particularly at the rubber-steel interface of each laminate and around any holes in the laminates. Techniques are needed to predict the axial stiffness, in both tension and compression, for bearings with high shape factors and any variation of this stiffness with increasing shear strain. The change in overall height with increasing shear strain has to be established as has the effect of creep on both axial deformation and stress or strain relaxation within the elastomer. Methods of establishing both the critical displacement at which rollover is imminent and the critical buckling load at high shear strain are needed. The use of such mechanical coupling of the bearing to the rest of the structure as is provided by dowels or bolts and the effects of these boundary conditions on other characteristics requires further investigation.

Physical testing of prototype bearings is clearly desirable however such programs tend to be expensive and the current requirement of verification testing of samples of production components could be reduced significantly if better substantiated mathematical models were available.

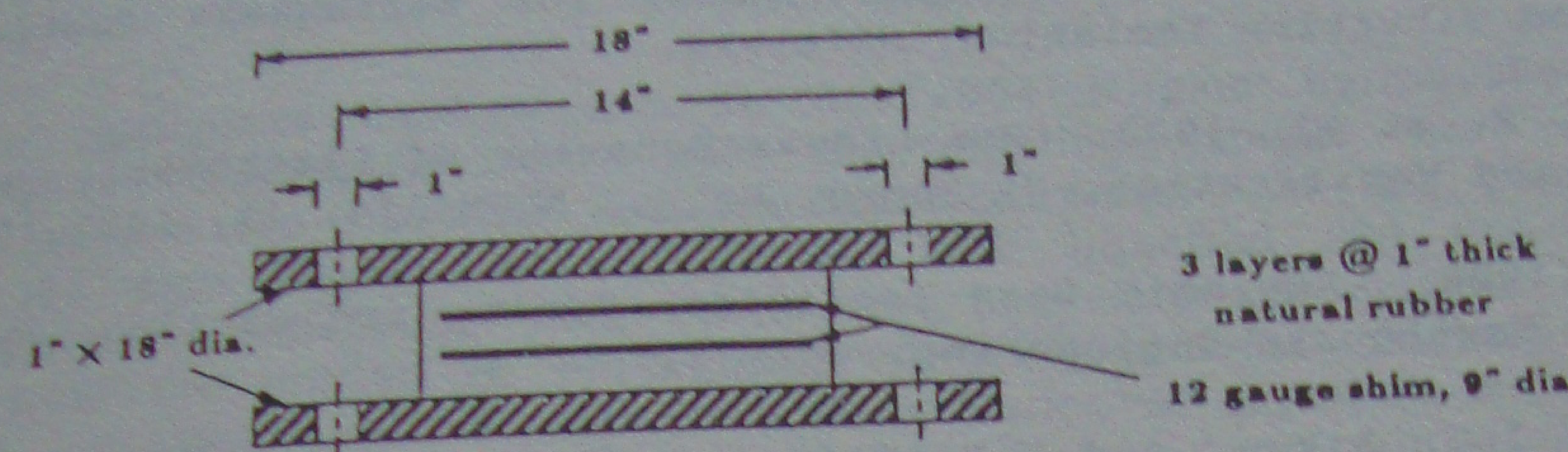
Some closed form mathematical solutions which are available are limited in scope and are not directly applicable to the large shape factors and high aspect ratios that are typical of many base isolators. Non-linear finite element models appear to offer considerable promise. Once a verified computer code is available a series of parameter studies is likely to provide improved understanding of the mechanics of elastomeric base isolators and thereby lead to improved design methodologies for these components.

ONGOING RESEARCH

A finite element model of an elastomeric bearing is being studied using PATRAN. This is a general purpose, 3-D Mechanical Computer Aided Design (MCAE) tool developed by PDA Engineering that allows finite element representation of an object before construction. It uses extensive graphics capabilities to aid in interpreting input and output data. PATRAN can link with virtually every major finite element analysis program, along with its own P/FEA finite element code.

Initially, version 2.3 of this coding was available on an Apollo Domain 3500. A low shape factor bearing was chosen for analysis using PATRAN and P/FEA since the discretization would not be excessive. This bearing, shown in Figure 1, was tested at U.C. Berkeley. The results were published by Tajirian, et al.(1990).

A 3-D, F.E.M. of the bearing was discretized with a total of more than 600 elements. Vertically, one element was used to represent the steel shims and end plates, while the rubber was represented using four elements. The rubber was assumed to be isotropic, linear elastic, and incompressible. Therefore, Poisson's ratio, ν, was assumed to equal 0.5. The published experimental results were used to evaluate the rubber's shear modulus G. G was set equal to 120 psi, and with ν = 0.5, the elastic modulus, E, was calculated to be 360 psi. It was established that if ν was adjusted to 0.49983, close to the assumed 0.5, PATRAN's results for a linear, static analysis matched the experimental results very closely.



Quarter Scale Bolted Bearing (Tajirian, F., et al. 1990)

The availability of a DEC 5000/200 system together with the release of PATRAN's latest version 2.4 has allowed larger element meshes to be undertaken. Currently, the mesh includes 9 elements in each rubber layer, and Poisson's ratio was changed to 0.49998 yielding results that correspond to the experimental results. However, the finer mesh does affect the internal stress and strain values and distribution. This was expected since the P/FEA coding does not allow for large strain theory. The finer mesh also reduced the vertical stiffness of the bearing, hence Poisson's ratio was increased until the experimentally derived stiffness was obtained.

It is concluded that the P/FEA coding is valuable for obtaining an approximate solution to the internal stress and strain distribution. However, more accurate solutions are possible with other analysis codings. It is planned to substitute P/FEA with MARC analysis which is specifically tailored toward elastomeric materials. It is expected that analyzing the above bearing will result in some realistic stress strain distributions. If the analysis is successful, it is intended to model larger bearings with higher shape factors.

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